Programming Parallel Computers

Jukka Suomela · Aalto University · ppc.cs.aalto.fi

Part 6A: Designing parallel algorithms

Three concepts

- ***Computational problem***
  - specifies what we want
  - e.g.: **sort *n* numbers**
- ***Algorithm*** that solves it efficiently
  - tells how to solve it, on a somewhat abstract level
  - e.g.: **quicksort**
- Efficient ***implementation*** of the algorithm
  - actual C++ code that works well on real computers
  - e.g.: **std::sort** implementation in the GNU C++ Library

Three concepts

- ***Computational problem***
  - specifies what we want
  - e.g.: **sort *n* numbers**
- ***Parallel algorithm*** that solves it efficiently
  - tells how to solve it, on a somewhat abstract level
  - e.g.: **parallel quicksort**
- Efficient ***parallel implementation*** of the algorithm
  - actual C++ code that works well on real computers
  - e.g.: **__gnu_parallel::sort**

Three concepts

- ***Computational problem***
  - specifies what we want
  - e.g.: **sort *n* numbers**
- ***Parallel algorithm*** that solves it efficiently
  - tells how to solve it, on a somewhat abstract level
  - e.g.: **parallel quicksort**
- Efficient ***parallel implementation*** of the algorithm
  - actual C++ code that works well on real computers
  - e.g.: **__gnu_parallel::sort**

Independent operations, opportunities for parallelism

Caches, registers, ILP, AVX, OpenMP, CUDA …

We need new kinds of algorithms

- Some classical algorithms have opportunities for parallelism
  - example: many "divide and conquer" algorithms
- However, often we need to design entirely new algorithms!
- Wrong question:
  "how to implement this algorithm on a parallel computer?"
- Right question:
  "how to design a parallel algorithm for this problem?"

Parallel algorithms: terminology

- ***"Processor":***
  - any form of parallelism often is described as if we had *p* processors
  - abstraction — **shows what can be done independently in parallel**
  - practical realizations: superscalar execution, pipelining, CPU vector lanes, CPU threads, GPU threads, multiple GPUs, computing cluster …
- ***"Work":*** total number of operations by all processors
- ***"Depth":*** longest sequential dependency chain
  - how long does it take even if we had infinitely many processors

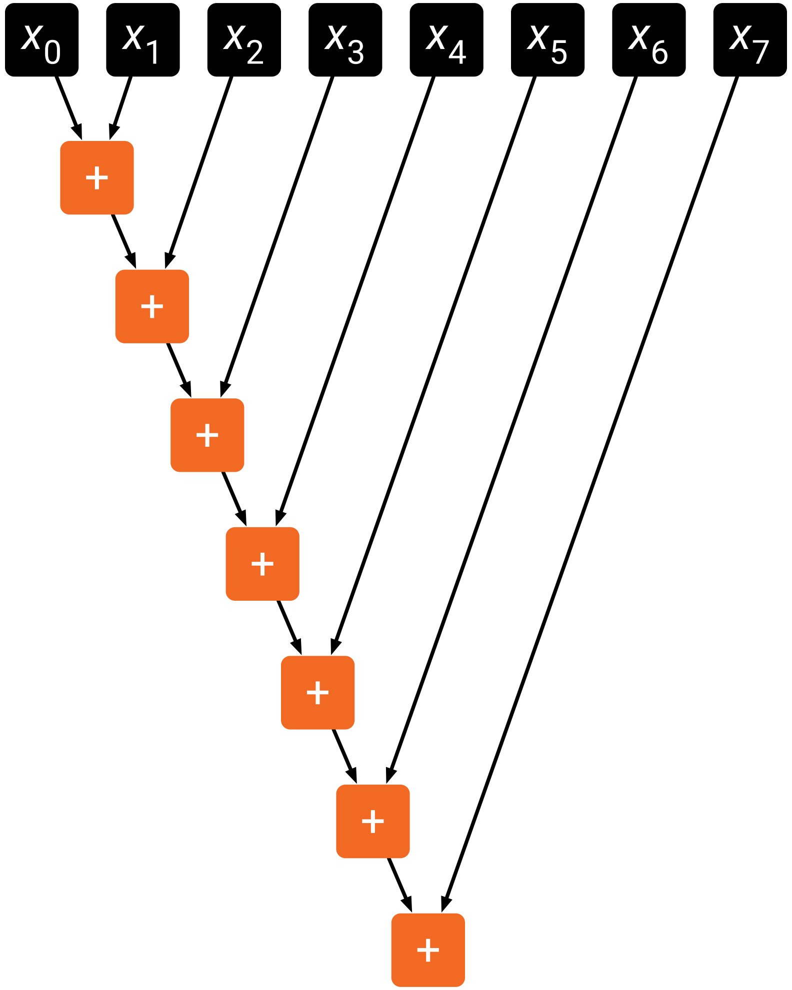
Sum

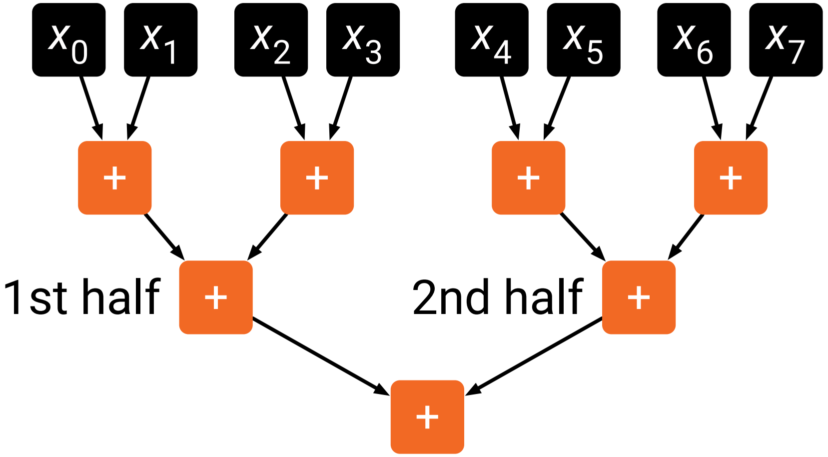
- Problem: calculate sum of $X = (x_0, x_1, \ldots, x_{n-1})$
- Trivial sequential algorithm
- Recursive parallel algorithm **sum(*X*)**:
  - if $n \leq 2$:
    - use sequential algorithm
  - if $n > 2$:
    - split X in two halves A and B
    - ***in parallel***, calculate a = **sum(*A*)** and b = **sum(*B*)**
    - return $a + b$

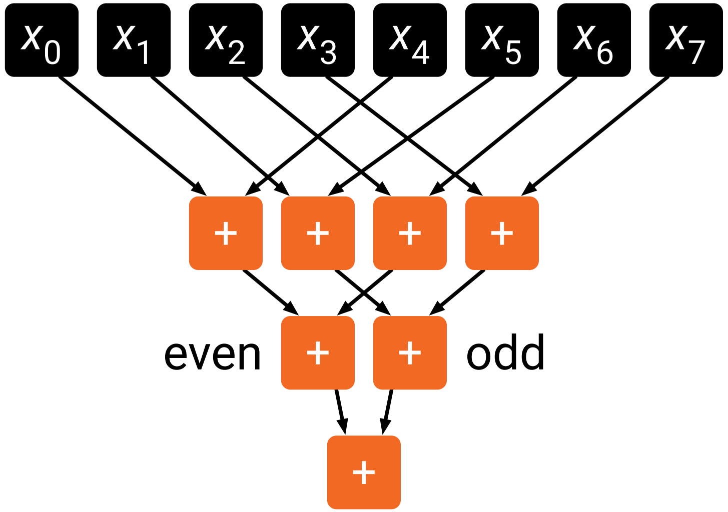
Some examples:

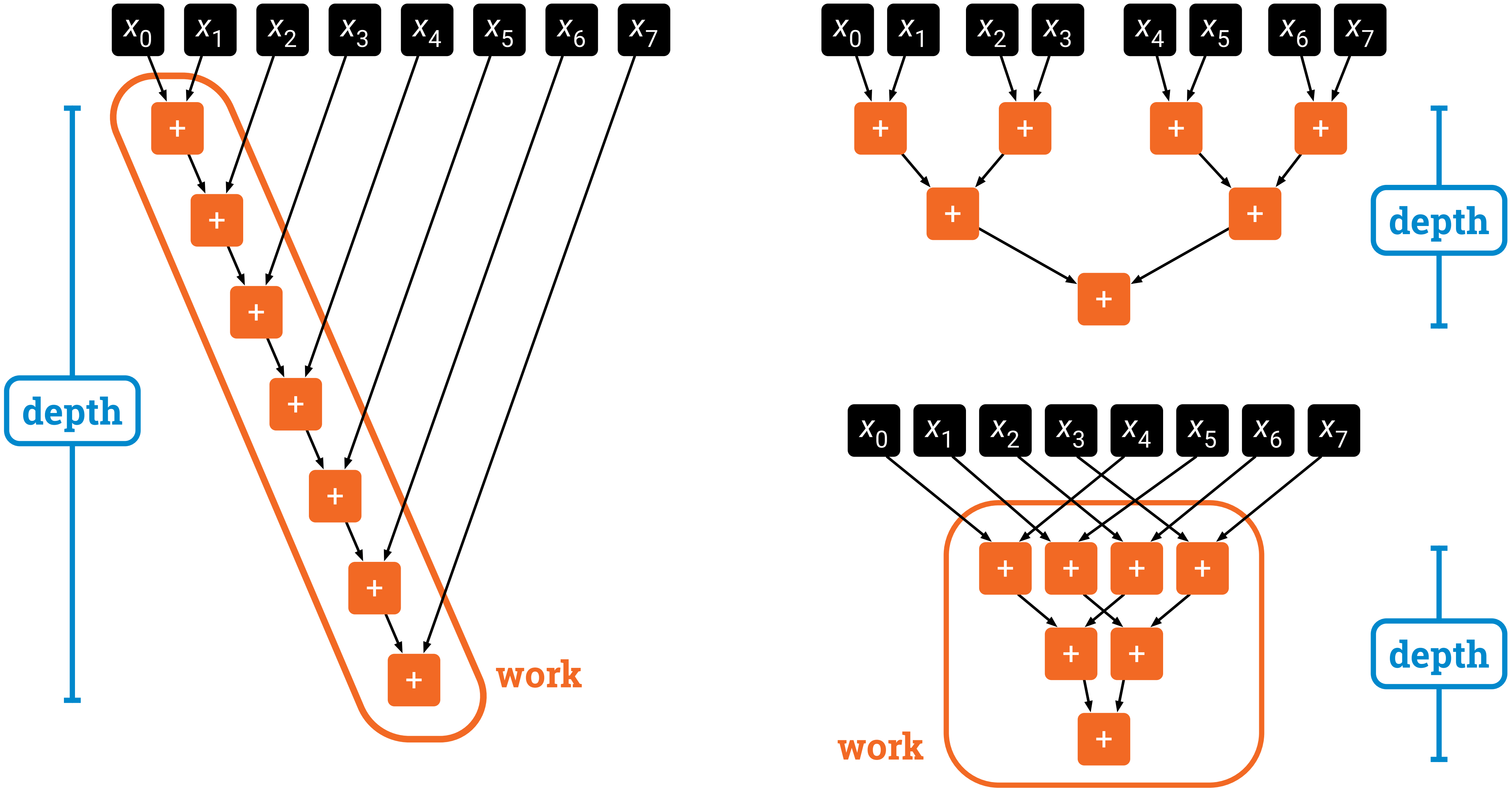
A = first half
B = second half

A = odd indexes
B = even indexes


x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7
+

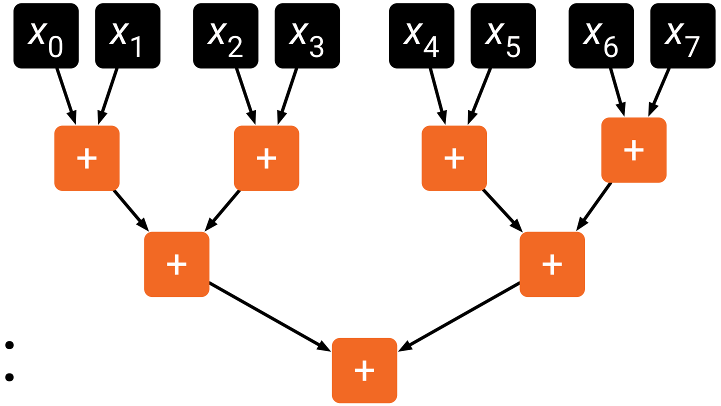

x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7
+
1st half
2nd half


x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7
+
even
odd



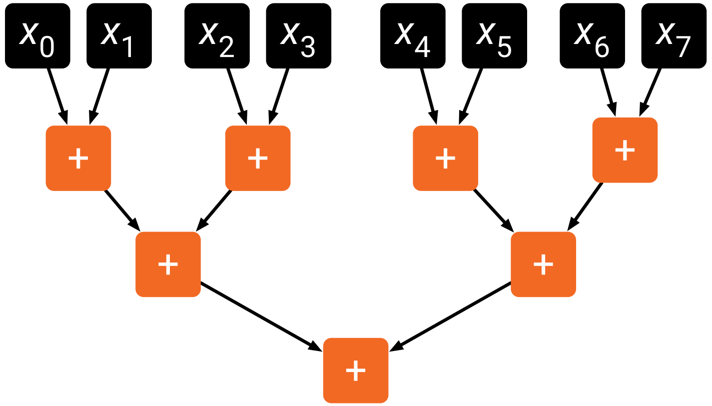
x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7
depth
work
x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7
depth
x_0 x_1 x_2 x_3 x_4 x_5 x_6 x_7
depth
work

Sum



- *In theory* we could parallelize sums as follows:
  - $O(n)$ processors, $O(n)$ work, $O(\log n)$ depth
- *In practice* this shows that there is **lots of potential for parallelism**, without doing much extra work
  - *do not* try to implement the recursive algorithm directly, use it as a source of ideas of how you could reorganize computation
  - just use enough levels to fully utilize your hardware
    - e.g.: 3 levels for OpenMP, 3 levels for SIMD, 2 levels for ILP?
  - usually we don’t have *n* “processors” but only e.g. 256

Sum



- The same idea works for any ***associative binary operation***:
  - sum
  - product
  - max
  - min
  - bitwise and, or, xor
  - matrix multiplication …

What can be parallelized?

- Nobody knows yet!
- Efficient parallel algorithms exist for many problems
- Some evidence that some problems are very hard to parallelize
    - some useful keywords for further study: complexity class **NC**, **P-complete** problems, conjecture **P ≠ NC**

Next

- Part 6B: ***parallel prefix sum*** — a concrete example of an efficient parallel algorithm
- Part 6C: ***pointer jumping*** — a useful algorithm technique for parallel algorithms that handle linked data structures